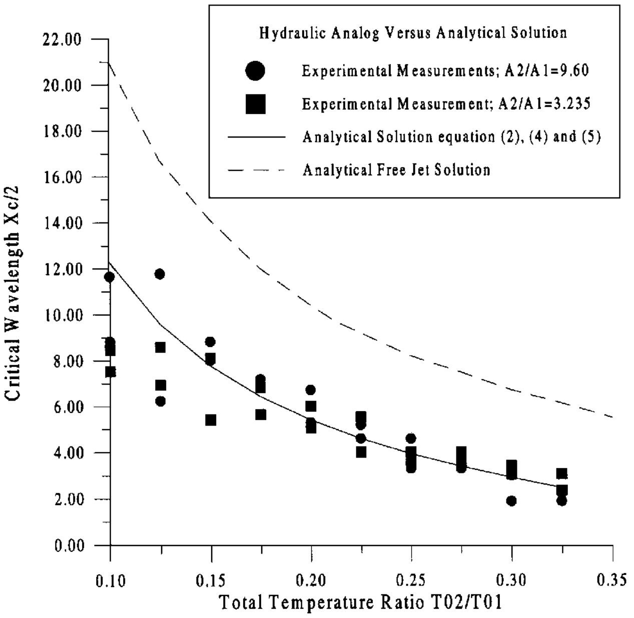

Fig. 3 Critical interface lengths.

Comparison of Model with Experimental Data

The key parameter x_c has been measured using the hydraulic analogy described by De Chant and Caton[7] and is presented in Fig. 3 for several area ratios and a Mach 2.0 primary stream interacting with a negligible secondary flow with fixed secondary total pressure and temperature. The area ratios associated with the measurements are large enough so that the pressure matching closure is suitable. Indeed, for the large area ratios considered, both the $A_2/A_1 = 9.60$ and $A_2/A_1 = 3.235$ analytical curves are virtually identical. As such, only a single combined curve is presented. So as not to leave the reader with the impression that this analysis is independent of area ratio, a freejet solution,[2] i.e., $A_2/A_1 \rightarrow \infty$ is presented in Fig. 3 for comparison. It is apparent that the freejet wavelength is a poor approximation to the internal flow interface solution.

Additionally, to help assess the effect of experimental error, we ran a series of repeatability tests, using different reservoir ratios to assess the effect of initial reservoir ratio on the measurements. A significant but acceptable amount of error is introduced by varying this ratio, especially significant differences in reservoir height, i.e., T_{02}/T_{01} small, because three-dimensional effects are pronounced for these small total temperature ratios.

Conclusions

We have developed and compared to experimental hydraulic analog data inviscid, confined supersonic and subsonic stream interactions. Formulas for the first critical location of the slipline are derived. Reasonable agreement is shown between the extended, internal flow, theoretical model and experimental measurements. This work is of considerable interest to aerodynamic mixer ejector nozzle development, a potentially critical technology to the high-speed civil transport program.

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k–ζ (Enstrophy) Compressible Turbulence Model for Mixing Layers and Wall Bounded Flows

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Introduction

THE objective of this work is to extend the k–ζ model of Robinson et al.[1] to compressible turbulent flows. The success of the k–ζ in reproducing a variety of incompressible flows should provide a good basis for the current model. As in the incompressible model, the current model will be based on the exact compressible equations to ensure that the correct physics is incorporated.

Compressible flows require for their description a velocity field and two thermodynamic variables, such as the density and temperature. Because of this, the fluctuations of these thermodynamic variables are as important as those of the velocity in determining the resulting turbulent flow. As a result, traditional two-equation and stress models[2] have proven to be inadequate in describing such flows. Therefore, it appears that an appropriate compressible turbulent flow model of the two-equation variety should include six equations that describe variances of velocity, density, temperature, and their respective dissipation rates.

A simplification of this approach can be achieved by using Morkovin's hypothesis.[3] According to this hypothesis, the pressure and total temperature fluctuations are small for nonhypersonic compressible boundary layers with conventional rates of heat transfer, i.e.,

$$p'/P \ll 1, \qquad T_0'/T_0 \ll 1 \tag{1}$$

As a result,

$$\rho'/\rho \simeq -T'/T \simeq (\gamma - 1)M^2(u'/U) \tag{2}$$

In this equation, p', T_0', ρ', T', and u' are the fluctuating pressure, total temperature, density, temperature, and velocity, respectively. Also, M is the Mach number and γ is the ratio of specific heats. The remaining variables represent mean properties. Based on Eq. (2), equations governing variances of turbulent quantities can be taken as the equation for the turbulent kinetic energy. However, equations governing the dissipation rates of the resulting variances may not be the same.

The approach employed in developing a k–ζ compressible model is to use the guidelines set in Ref. 1 to model the additional terms. Thus, for low Mach numbers, the new model reduces to that of Ref. 1. It is shown in Ref. 4 that when Morkovin's hypothesis is invoked, the effects of compressibility for flows considered here result in an additional term in the k equation, which is

$$-C_1(\rho k/\tau_\rho) \tag{3}$$

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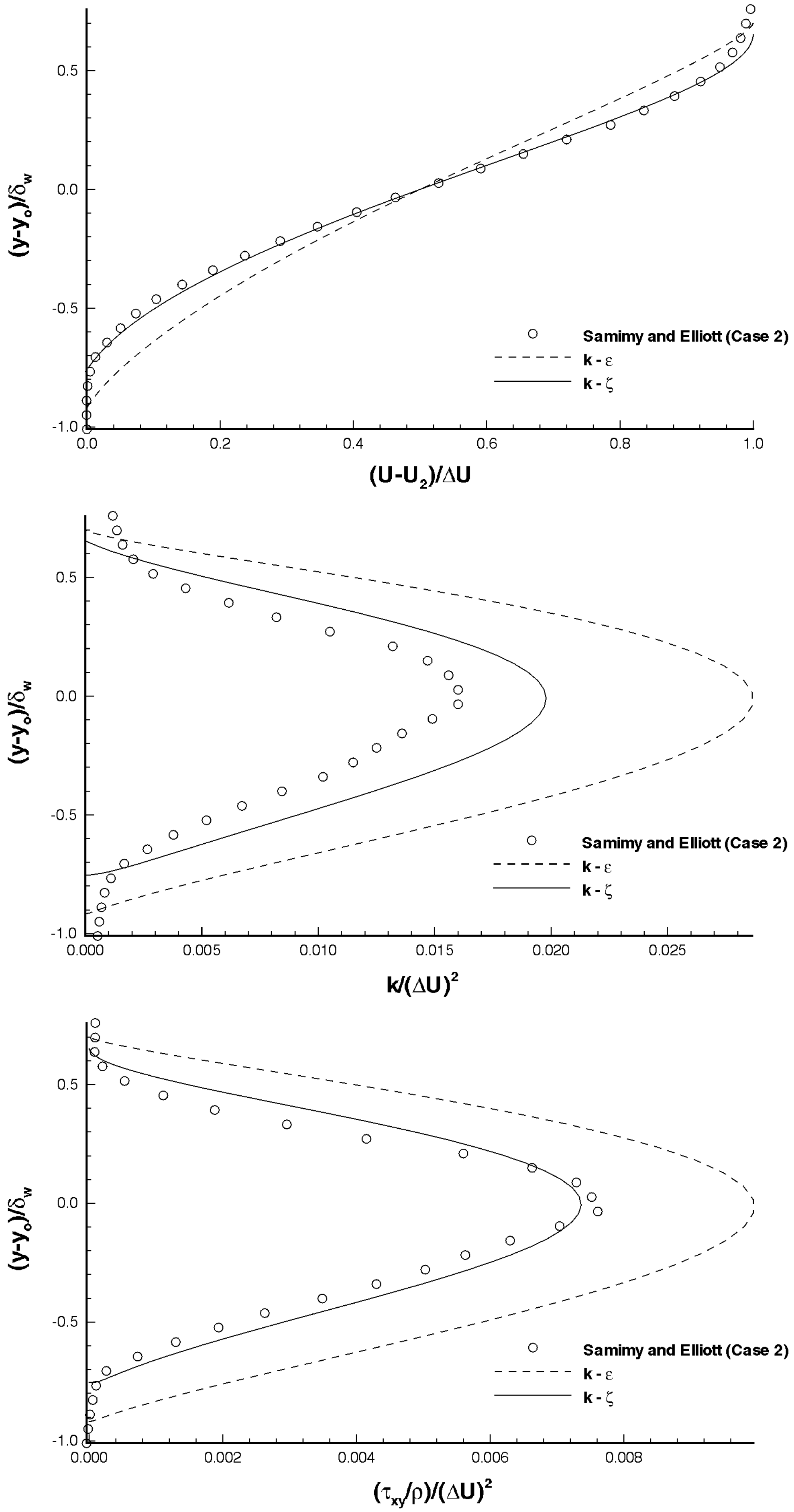

Fig. 1 Turbulence model comparisons with experiment of Samimy and Elliott[7] case 2.

where

$$\frac{1}{\tau_\rho} = \frac{1}{\rho}\sqrt{\frac{\partial \rho}{\partial x_i}\frac{\partial \rho}{\partial x_i}k} \tag{4}$$

Similarly, the term

$$-C_{\zeta_1}(\sqrt{2\zeta}/\tau_\rho \Omega)\Omega_i \tag{5}$$

is added to the ζ equation. In the preceding equations, ρ is the density, Ω_i is the vorticity vector, and C_1 and C_{ζ_1} are model constants given by 0.6 and 1.35, respectively.

It is generally accepted that the k–ω model outperforms the k–ϵ model for wall bounded shear flows whereas the reverse is true for free shear layers.[5] Therefore, the present model is compared with the k–ϵ model when free shear flows are considered and with the k–ω model when wall bounded flows are considered.

Results and Discussion

To validate the present model, comparisons were made with various experimental measurements of compressible mixing layers and wall bounded flows.[4] The ability of the model to calculate compressible mixing layers will be illustrated first. Settles and Dodson[6] tabulate and plot (see Table II and Fig. 1 of Ref. 6) normalized mixing layer growth rates vs convective Mach numbers. The data are collected from various sources. As may be seen from the data and figure, there is a great deal of scatter rendering such tabulations and plots meaningless. As a result, there is no acceptable growth rate for a given convective Mach number. This may be contrasted with the incompressible case where growth rates are well documented. The reason for the scatter can be traced to a lack of uniformity in calculating and defining growth rates. Moreover, there is a lack of uniformity in defining the width of the mixing layer. Because of this, judging the worth of a compressible turbulence theory solely on its prediction of growth rates is somewhat misleading. Thus, rather than restrict our comparisons to growth rates, we opted to compare the predictions of the theory with the measured velocity, turbulent kinetic energy, and turbulent shear stress. Because growth rates are determined from velocity profiles, good agreement with measured velocity is a good prediction of growth rate, independent of the way it is defined.

The resulting system of governing equations is a total differential system and was solved by a method identical to that of Ref. 1. Comparison of present results with the k–ϵ model and experiment is shown in Fig. 1. In presenting the data, we followed the suggestion of Ref. 7 and scaled y with δ_ω, the vorticity thickness, defined as

$$\delta_\omega = \Delta U \bigg/ \left(\frac{\partial u}{\partial y}\right)_{\max}, \qquad \Delta U = U_1 - U_2 \tag{6}$$

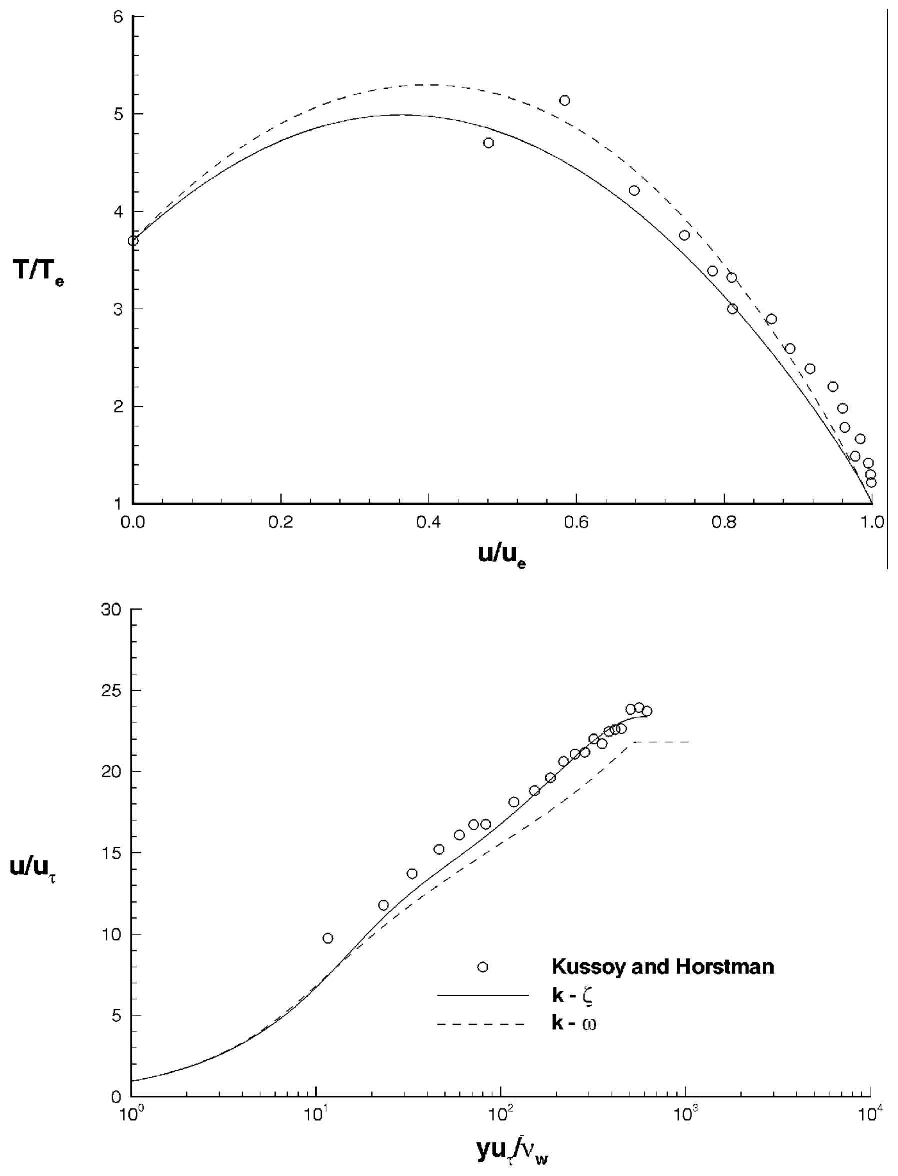

Fig. 2 Comparison of calculated mean temperature–velocity distribution and law of the wall vs experiment of Kussoy and Horstman.[8]

Figure 1 compares the predictions of the present theory and that of the k–ϵ model with the Wilcox compressibility correction[5] with case 2 of Ref. 7. For this case,

$$U_2/U_1 = 0.25, \qquad \rho_2/\rho_1 = 0.58, \qquad T_{0_1} = T_{0_2} = 276\ \text{K}$$
$$M_1 = 1.96, \qquad M_2 = 0.37, \qquad M_c = 0.64 \tag{7}$$

where M_c is the convective Mach number. It is seen that the present model outperforms the k–ϵ model. Because the present model makes use of Morkovin's hypothesis, it is expected that its predictions will deviate from measurements at high convective Mach numbers and high stagnation temperature ratios.

To illustrate the ability of the model to compute wall bounded flows, the work of Kussoy and Horstman[8] for a cold-wall boundary at a freestream Mach number $M_\infty = 8.18$ is selected. Calculations were obtained using a modification of the boundary-layer code by Harris and Blanchard.[9] Solutions provided by this code are second-order accurate. The freestream boundary conditions imposed on the model are $k_\infty/U_\infty^2 = \zeta_\infty/(U_\infty/\ell)^2 \sim 10^{-7}$, where ℓ is a characteristic length scale and was set equal to 1. The wall boundary conditions are $k_w = 0$, and ζ_w was set such that $k \sim k_0 y^2$ with molecular diffusion balancing dissipation as required in the k equation. A grid of 175 points in the normal direction with geometric stretching of 7% was used in all boundary-layer calculations. A grid study was conducted, and it was determined that the results were grid independent. Comparisons with experiment and the k–ω model are shown in Fig. 2, where subscript e refers to edge conditions, u_τ is the friction velocity, and ν_w is the kinematic viscosity evaluated at the wall temperature. It is seen that the k–ζ model agrees well with experiment. Differences in measured skin-friction and heat transfer coefficients and predictions of the k–ζ model are 6.9 and 7.1%. Similar results for the k–ω model are 8.1 and 19.2%.

In conclusion, the present model outperforms both the k–ϵ and the k–ω models. Thus, we have at hand a model that is capable of calculating low-speed and high-speed flows with one set of model constants.

Acknowledgments

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Motion of a Body Through Large-Scale Inhomogeneity in the Stratified Atmosphere

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Introduction

BODY motion in the atmosphere is essentially a coupled problem: the body motion depends on aerodynamic coefficients, which in turn are found by solving the problem of the flowfield around the body. The input data for the aerodynamic problem are parameters of the incident flow, body geometry, and spatial orientation of the body.

To solve this problem with a high degree of accuracy, one should integrate simultaneously both the equations of motion of the body and the equations describing the flowfield around the body. This approach requires considerable computer resources but can be improved in the case of free supersonic motion of a slender body of revolution. Such a motion is usually accompanied by small angles of attack.

We consider supersonic flight of a blunt slender body of revolution through a large-scale cloud of heated gas (thermal) floating in the stratified atmosphere. An efficient numerical method to determine the body motion is proposed. The dependence of the solution of the flow problem on the angle of attack is found analytically by using this method. The proposed method shows how the thermal environment alters the trajectory, spatial orientation, and flight stability of the body. How the location of the center of pressure depends on the Mach and Reynolds numbers of the incident flow has also been studied.

Statement of the Problem

We will assume that, at an instant of time in the stratified atmosphere, an axisymmetric cloud of heated gas is formed having the following parameters:

$$T(h, d) = T_a(h) + [T_{\max} - T_a(h)] \exp\left[-\left(R_T^{-1} R\right)^2\right]$$

where $R_T = 1.2$ km, $T_{\max} = 1600$ K, $T_a(h)$ is the temperature of the unperturbed atmosphere at the altitude h, d is the distance from the thermal's axis of symmetry, $R = [(h - H)^2 + d^2]^{1/2}$, and $H = 20$ km. The position of the temperature maximum $T_{\max}$ corresponds to the altitude H.

The cloud floats up and forms a vortex ring. The cloud begins to float up at the altitude of 22 km, and 15 s later the body enters it horizontally with a velocity $V_0 = 2000$ m/s (the vertical component of the velocity is equal to zero). The body has the form of a cone with a blunt spherical nose of radius $R_0 = 0.1$ m, semivertex angle of 15 deg, length $L = 2$ m, and mass of 1000 kg; the center of mass of the solid is situated a distance L_c from the spherical nose. Initially, while the thermal does not influence the body motion, the trajectory plane is 500 m from the axis of symmetry of the thermal. The relative position of the body and the thermal at the initial instant of time is shown in Fig. 1.

At the instant the body enters, the gas is twisted into a toroidal vortex ring, which floats up into the atmosphere. In view of this, the freestream flow has a varying space–time structure. We will assume that the body does not influence the gas motion in the thermal. The flow past the body is assumed to be laminar.

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